Revista Latinoamericana de Tecnología Educativa, 24(1)
https://doi.org/10.17398/1695-288X.24.1.77

RELATEC
Revista Latinoamericana de Tecnología Educativa

ARTICLE / ARTÍCULO

The Use of GeoGebra in Teaching Mathematical Concepts: Practices, Barriers, and Teachers' Perceptions

El uso de GeoGebra en la enseñanza de conceptos matemáticos: prácticas, barreras y percepciones docentes

Received: July 15, 2024
Accepted: December 17, 2024

Author's address:

[1,2] Departamento de Didáctica de las Matemáticas y de las Ciencias Experimentales, Universidad Internacional de La Rioja (UNIR), Avenida de la Paz, 137, 26006 Logroño, La Rioja (Spain)

[3] Departamento de Matemáticas. Área de Didáctica de la Matemática. Facultad de Educación. Universidad de Zaragoza. Calle de Pedro Cerbuna, 12, 50009 Zaragoza (Spain)

E-mail / ORCID

luis.dubarbie@unir.net

https://orcid.org/0000-0001-9133-1128

alvaro.barreras@unir.net

https://orcid.org/0000-0001-5325-8505

oller@unizar.es

https://orcid.org/0000-0002-8191-3199

Luis Dubarbie-Fernández[1], Álvaro Barreras[2] & Antonio M. Oller-Marcén[3]

Abstract: The use of GeoGebra in teaching mathematics has grown significantly in recent years, so this study aims to explore the use of this software by teachers in teaching the limit of a function. For this purpose, a questionnaire was designed and validated and has been answered by 129 mathematics teachers. The data were analysed both quantitatively and qualitatively with the aim of discerning obstacles in the use of GeoGebra and the development of ad hoc applets, identifying the most outstanding features of GeoGebra applets for teaching the limit concept and determining the stages in the teaching-learning process at which this tool is used. Thus, a limited use of GeoGebra in teaching the limit concept and a very low percentage of teachers developing their own applets has been identified, mainly due to a lack of technological resources and lack of knowledge. Furthermore, interactivity and the possibility of using various systems to represent the limit concept are revealed as the characteristics most highly valued by teachers. Finally, GeoGebra is used most to give examples but rarely used in the evaluation process.

Keywords: GeoGebra, Educational Technology, Mathematics Teachers, Mathematical Concepts, Mathematics Curriculum, Secondary Education.

Resumen: La utilización de GeoGebra en la enseñanza de las matemáticas ha crecido notablemente en los últimos años, por lo que en este estudio se pretende indagar acerca del uso de este software por parte del profesorado en la enseñanza del límite de una función. Para ello, se ha diseñado y validado un cuestionario que ha sido respondido por 129 docentes de matemáticas. Los datos han sido analizados tanto cuantitativa como cualitativamente con el objetivo de conocer obstáculos para el uso de GeoGebra y para la elaboración de applets propios, identificar las características más destacadas de los applets de GeoGebra para la enseñanza del límite y determinar los momentos del proceso de enseñanza-aprendizaje en los que se utiliza esta herramienta. Así, ha sido identificado un escaso uso de GeoGebra en la enseñanza del límite y un bajo porcentaje de docentes que elaboran sus propios applets debido, fundamentalmente, a la ausencia de recursos tecnológicos y a la falta de conocimientos. Además, la interactividad y la posibilidad de utilizar varios sistemas de representación del límite se revelan como las características más valoradas por los docentes. Finalmente, GeoGebra es especialmente utilizado a la hora de mostrar ejemplos y apenas se usa en el proceso de evaluación.

Palabras clave: GeoGebra, Tecnología Educativa, Profesorado de Matemáticas, Conceptos matemáticos, Curriculum de Matemáticas, Educación Secundaria.

1. Introduction

The notion of limits is highly important from a mathematical perspective. A limit is a complex object commonly found in advanced mathematical thought (Tall, 1991), which has received a lot of attention in research on mathematics education. Furthermore, numerous research projects have explored the potential of using dynamic and interactive environments for teaching the limit concept (Martinovic and Karadag, 2012). At present, GeoGebra is one of the most widespread dynamic and interactive mathematics learning environments (Hohenwarter et al., 2009). Within the context of learning the limit concept, the use of GeoGebra could help students to overcome certain significant obstacles linked to this concept (Rodríguez et al., 2020). Hutkemri (2014) also notes that this tool enables students to enhance their conceptual and procedural knowledge about the notion of limits, but that it "is necessary to provide training for teachers on the advantages of GeoGebra and its operating skills" (p. 880). To provide this training, a clear vision is needed of the features of the available applets in official GeoGebra repositories (Barreras et al., 2022), but it is also important to understand teachers' practices when using this software in situations involving teaching-learning of the limit concept.

In this context, the following general research question can be raised: How often, why and in what way do teachers use GeoGebra when teaching and learning the limit concept? This study aims to advance, at least partially, in answering this research question. In particular, the following specific objectives have been set:

1. Determining how often teachers use GeoGebra in teaching-learning processes involving the limit concept and identifying possible obstacles.
2. Discerning how often ad hoc GeoGebra applets are designed and possible obstacles to doing so.
3. Examining how useful GeoGebra is in teaching-learning processes involving the limit concept.
4. Determining the points in the instruction process at which GeoGebra is used.

1.1. Literature review

Martinovic and Karadag (2012) note that dynamic and interactive mathematics learning environments provide an environment of experimentation of great pedagogical value, offering students the possibility to explore different mathematical concepts and the relationships between these concepts so that they can develop their own cognitive frameworks. In particular, these authors assert that «the perception of constant change in mathematical objects may affect students' understanding of these concepts, leading them to develop a new type of learning.» (p. 47).

Thus, the use of GeoGebra for teaching differential calculus —and the limit of a function in particular— may help students gain a better understanding of the mathematical concepts discussed. Among other factors, this is due to the interactivity of applets, their dynamic nature (Sari, 2017) and the possibility of working simultaneously with diverse systems of representation (Caligaris et al., 2015). In relation to this, Barreras et al. (2022) analyse the use of GeoGebra for teaching the limit concept, focusing on several variables: interactivity, conceptual image, systems of

representation and actions. In their work, they conclude that these variables should be interconnected when it comes to using this software, and therefore, interactivity should be one of the resources for combining different systems of representation, so as to foster efficient conceptual images of the limit.

Conceptual aspects aside, one of the matters to bear in mind when integrating GeoGebra into learning situations is the way in which the tool can contribute to the development of different mathematical processes in students (NCTM, 2000). In this regard, it has been found that GeoGebra may offer sufficient support to achieve a decent level of development in mathematical skills related to processes of concept representation, problem solving, reasoning and proof, making connections and communication (Romero et al., 2015).

In addition, GeoGebra prompts students to play a more active role in their learning and enables teachers to design more effective learning situations, promoting a two-way process of teaching and learning (Hutkemri, 2014). In general, it has been found that conceptual aspects related to functions and their representation are learned more effectively when GeoGebra is used, even when working in groups, due to the possibilities for exploration and construction. The reason for this is that students can devote more time to analysing connections between mathematical concepts than to doing calculations. Furthermore, GeoGebra also enables students to recognise whether they have learned correctly and to detect errors themselves (Takaci et al., 2015). However, certain attitudes, such as creativity or reasoning and proof, may develop in a smaller number of students, so the teacher must act in real time to orchestrate individual dynamics (García et al., 2021, p. 194). The interactivity and browsability of GeoGebra foster the development of autonomy in students, boosting their flexible thinking capability in problem solving. In addition, GeoGebra offers students the option of making their own representations stemming from decisions and strategies they come up with themselves which, with the aid of guided questions from the teacher, increases the level of precision and rigour with which they can work in class and the development of students' reasoning capabilities (Romero and García, 2023).

However, the teacher's role when using GeoGebra in the classroom must be assessed in order to ensure that the benefits that this tool can afford are actually achieved (Iranzo and Fortuny, 2009). In fact, as Arnal-Bailera and Oller-Marcén (2020) have noted, low levels of training in the use of GeoGebra can lead to misuse of the tool. This entails the need to add the use of GeoGebra in teacher training, so that they integrate knowledge stemming from diverse domains (pedagogical knowledge, content knowledge and technological knowledge), as Koehler et al. (2013) propose.

The use of GeoGebra in teacher training programmes was recently found to be beneficial in the assimilation and application of mathematical concepts (García-Lázaro and Martín-Nieto, 2023) and in the visualisation of geometric concepts (Dockendorff and Solar, 2016) in future teachers and in students (Guarin and Parada, 2023). The tool is also effective when it comes to improving attitudes of teachers in training towards mathematical demonstrations (Zengìn, 2017b). In fact, Zengìn (2017a) believes that teachers in training should become acquainted with this software because of the benefits offered in developing their mathematical communication capabilities. In turn, when active teachers participate in training programmes on the use of GeoGebra, these teachers then take a more student-focused approach to their teaching (Marange and Tatira, 2023).

At any rate, the ways in which teachers actually use this software before or after the training process, and the reasons for doing so, must be examined. Lasa and Wilhelmi (2013) describe three possible stages at which a teacher may integrate the use of GeoGebra in the classroom in relation to geometry: exploratory (constructing models for solving exercises and problems or for inferring properties, illustrative (giving examples of properties based on specific cases) and demonstrative (proving and demonstrating inductive and deductive properties). These authors also discuss the potential of this software for coordinating the work done at these three stages. Other research (Carvalho et al., 2023; Rosyidi et al., 2024) has also described the possibility of integrating the use of GeoGebra at the evaluation stage.

In addition, McCulloch et al. (2018) describe the reasons why teachers integrate the technology in the classroom: generating opportunities to enhance conceptual or procedural comprehension, avoiding mistakes in performing routine tasks, giving meaning to ideas and implementing mathematical procedures. They also note that the factors involved in choosing a specific technology instrument include aspects such as ease of use, access options, interactive features and potential for enhancing the instruction process. Saralar-Aras (2022), in turn, points out motives related to visualisation, enhanced student learning, the increase in student motivation and the decrease in teacher workload. This author also describes obstacles such as classroom management and lesson planning. However, integrating GeoGebra poses certain challenges related to issues that go beyond teacher training per se, such as access to technological resources and students' difficulties in using the tool (Wassie and Zergaw, 2019).

2. Method

To achieve the specific objectives described above, an exploratory and descriptive research approach was used. The methodological design coincides with what Creswell (2012) referred to as 'embedded design'. In this case, the design involved the use of quantitative primary data gathered from a questionnaire, which also contained open questions that provide qualitative secondary information to support and supplement the quantitative data.

2.1. Instrument

The questionnaire is divided into three sections. The first section contains 7 questions to provide context. The second section has 5 questions about general issues related to teaching-learning about the limit of a function. Finally, the third section contains questions about the use of GeoGebra by the surveyed teachers in teaching the function limit.

After drafting a preliminary version of the questionnaire, in order to verify its validity (Elangovan and Sundaravel, 2021), four researchers and university instructors in the area of didactics of mathematics, selected at the authors' discretion, were asked to assess it. These experts were consulted with regard to the clarity and pertinence of each of the questions, and were able to make remarks and suggestions in relation to the questions and overall design of the questionnaire. The following two criteria were used in relation to the experts' opinions:

- If two or more experts indicated that a question was not pertinent, it would be removed from the questionnaire.

- If one expert deemed a question to lack clarity, the wording would be revised.

Following the experts' remarks, certain questions were added and the value of the Likert scales was also clarified, adding verbal explanations to the numerical scores: 1 (not important), 5 (very important).

A pilot test of the second version of the questionnaire was conducted on ten Spanish secondary school mathematics teachers, also selected at the authors' discretion. These teachers filled out the questionnaire and were then asked about the clarity and pertinence of the questions. They were also asked about how useful the questions were for gathering information about the use of GeoGebra in teaching the limit. All the participants in the pilot test agreed that the wording of the questions was clear and that all the questions were pertinent. The suggestions received for improvements in the questionnaire dealt with matters unrelated to the research aims, so they were excluded from this study. The final version of the questionnaire can be found in Appendix I.

2.2. Sample

The questionnaire was sent by email, using the Survey Monkey tool, to a total of 514 teachers enrolled in a Master's in Didactics of Mathematics in Compulsory and Post-Compulsory Secondary Education. It was available from 24 October to 13 November 2023. The participation rate was approximately 25%, and responses were received anonymously from 129 mathematics teachers from several countries: Colombia (57.4%), Ecuador (38%), Dominican Republic (2.3%), Uruguay (1.5%) and Mexico (0.8%). 55.8% of the participants were men and 44.2% women. The average age of the participants was 37.9, with standard deviation of 8.8 years.

According to their responses, these teachers have an average of 7.7 years of experience teaching mathematics in secondary school (students from 12 to 18 years old). In this regard, inexperienced teachers are deemed to have 2 or fewer years of experience (OECD, 2013) while expert teachers are those with more than 10 years of teaching experience (Huang and Li, 2012). Therefore, the sample is composed of 34 participants with little experience, 35 teachers with extensive experience and the remaining 60 with average experience.

Finally, the figure below shows the formal education of the respondents, except for 8 responses (6.2%) which were too ambiguous to classify.

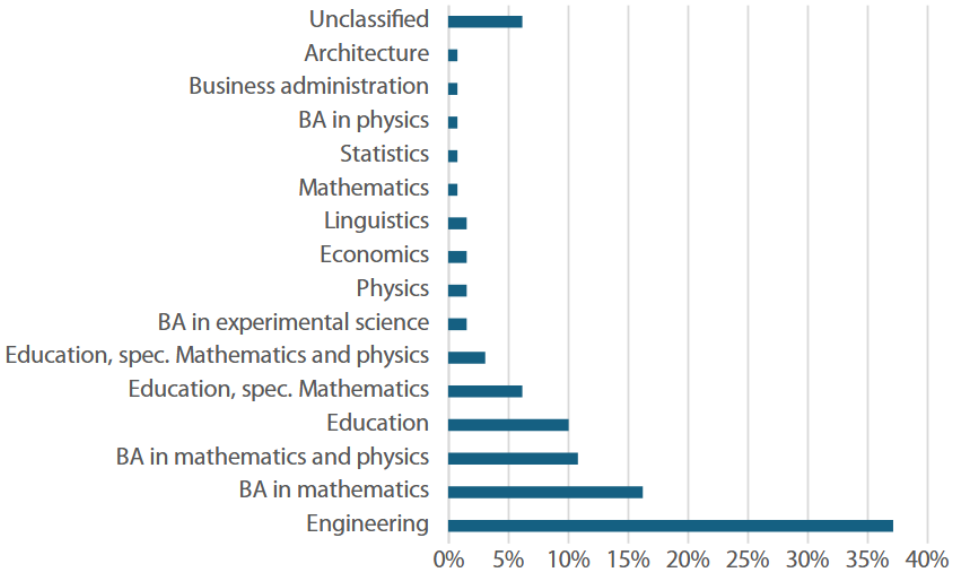


Figure 1. Formal education.

2.3. Analysis

To analyse the quantitative data collected in the questionnaire, statistical tools of an essentially descriptive nature were used, bearing in mind the ordinal numbers of the Likert scales used in some of the questions (Blaikie, 2003). Furthermore, in line with the research objectives described above, the questionnaire also included certain questions in which the participants had to choose between several options and others with an open response.

In relation to the first objective, the participants that had not used GeoGebra to teach the limit concept were asked to indicate the reasons in an open question, conducting an inductive analysis of the contents of their responses. The categories that emerged from this process are shown in Table 1.

To address the second objective, the participants were given the following list of reasons why they may have decided not to design their own GeoGebra applets (González Pérez and De Pablos Pons, 2015): Lack of knowledge, lack of time, use of applets available over the internet and others (filling in the reasons).

Table 1. Categories for analysis of reasons why GeoGebra is not used.

Categories	Description	Example
Lack of technological resources	GeoGebra is not used to teach the limit concept because the schools or students lack the required technological resources.	ID4: «We do not have technological resources to use applications at the institution.»
Lack of opportunity	GeoGebra was not used to teach the limit concept due to a lack of professional experience or not having taught this unit.	ID13: «Because I have not had a chance to teach this unit.»
Lack of knowledge	GeoGebra is not used to teach the limit concept because of a lack of knowledge about how it works.	ID42: «Lack of awareness about how the tool works.»
Out of didactic considerations	GeoGebra was not used to teach the limit concept because it was not deemed suitable from a didactic perspective.	ID45: «Because of the time it takes to explain to the secondary school students how to use GeoGebra, the available manuals are not comprehensible to them.»

In relation to the third objective, the teachers who claimed to have used GeoGebra were first asked to rank from 1 to 5 the different features of the applets. The proposed features were: jointly addressing several conceptual aspects, combining several representation systems, interactivity and others (filling in the features). In turn, in an open question they were also asked to provide an explanation about the reasons why they considered it useful in teaching the limit concept. These responses were analysed based on three variables: didactic criterion, advantages of using GeoGebra and processes. The description of the different categories (which are not mutually exclusive) considered for each one can be found in Table 2. The first variable and its categories were generated in an emerging fashion. In relation to the second variable, the categories considered were taken mainly from the work of Barreras et al. (2022). Finally, for the third variable, the mathematical processes defined by the NCTM (2000) were taken into account.

Finally, in relation to the fourth objective, the participants who claimed to have used GeoGebra in teaching the limit concept were asked about the stages of the instruction process where it was used. The closed but not mutually exclusive options provided were: when introducing concepts, when giving examples, when solving exercises, when solving problems and in the evaluation. These categories aim to somehow encompass the most common teaching practices in mathematics classrooms (Perrin-Glorian, 1999).

Table 2. Variables and categories for analysing GeoGebra usefulness. Variable: Didactic criterion.

Categories	Description	Example
Teaching	The response includes aspects inherent to the teacher's practices.	ID94: «The opportunity offered by the application when it comes to graphing and visualising functions is quite significant in the teaching process.»
Learning	The response includes aspects inherent to students' practices.	ID9: «GeoGebra provides tools for making the concepts more comprehensible.»

Table 3. Variables and categories for analysing GeoGebra usefulness. Variable: Advantages of using GeoGebra.

Categories	Description	Example
Visualisation	Applets facilitate visualisation of the limit of a function.	ID48: «Graphic visualisation is crucial and GeoGebra makes graphic analysis much easier.»
Interactivity	Applets offer the possibility of interacting.	ID95: «It allows the student to interact with the limit concept, making the learning meaningful.»
Combination of representation systems	Applets offer the possibility to combine different systems for representing the limit of a function.	ID30: «It allows students to integrate algebraic and graphic aspects of the limit concept, there they can analyse and compare.»
Support in problem solving	Applets foster problem solving using the limits of functions.	ID49: «It motivates the student for meaningful learning, it is representative for analysing and verifying solutions to problems.»
Emotional or motivational aspects	The use of applets motivates students and/or prompts a positive attitude.	ID127: «Students are more motivated to learn when these tools are used.»
Understanding the limit concept	The use of applets facilitates an understanding of the concept of the limit of a function.	ID16: «It is a way for the kids to understand better, more easily and simply.»

Table 4. Variables and categories for analysing GeoGebra usefulness. Variable: Processes

Categories	Description	Example
Problem-solving	Possibility of developing mathematical problem solving in all kinds of contexts.	ID81: «It helps us solve complex problems quickly.»
Reasoning and proof	Possibility of developing mathematical demonstrations and doing reasoning, making guesses and assessing arguments.	ID112: «GeoGebra is a tool that makes it possible to guess and verify those guesses.»
Representation	Possibility of using different representations of mathematical concepts.	ID20: «A limit of a function can be clearly visualised.»
Connections	Possibility of creating connections between different mathematical ideas or with everyday life.	ID21: «Because it allows for interaction between new technologies and knowledge applied in real life.»
Communication	Possibility of expressing mathematical ideas, communicating those ideas and practicing mathematical language.	n/a

To analyse the participants' responses to the open questions (Tables 1 and 2), each of the three researchers did their own analysis. Afterwards, the individual classifications were compared in order to reach a consensus. This triangulation process (Flick, 2004), in which three researchers use the same records, boosts the internal validity and reliability of the research (Hernández et al., 2010).

3. Results

The contents of this section are broken down according to the four research objectives described in the introduction.

3.1. Use of GeoGebra

Out of the 129 teachers who answered the questionnaire, just 54 (41.9%) claimed to have used GeoGebra at some point in their professional career in processes of teaching-learning the concept of the limit of a function. The remaining 75 claimed not to have used GeoGebra at all in teaching the limit concept. Figure 2 shows the distribution of this variable according to the participants' level of experience. Although not statistically significant, a greater use of GeoGebra for teaching the limit concept is seen among teachers with extensive experience. Approximately 46% claimed to have used it, compared to 35.3% of teachers with little experience and 43.3% of those with average experience.

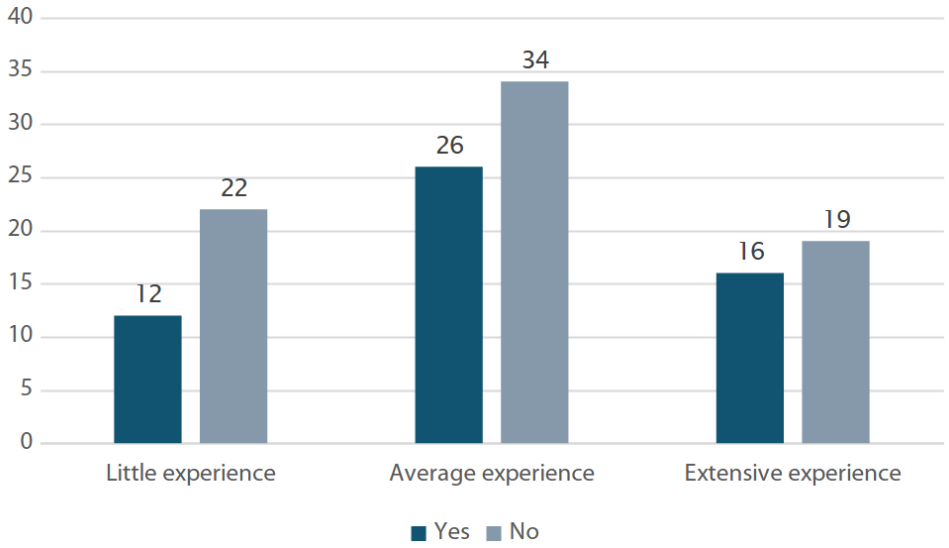


Figure 2. Use of GeoGebra according to level of experience

The reasons given by the teachers who claimed not to use GeoGebra to teach the limit concept were broken down into four categories (Figure 3): lack of technological resources (38.7%), lack of teaching experience or experience in teaching the limit concept (25.3%), lack of knowledge about the use of GeoGebra (24%) and didactic considerations, such as planning issues or student difficulties (8%). The others (6 teachers) did not give a reason or the response could not be classified.

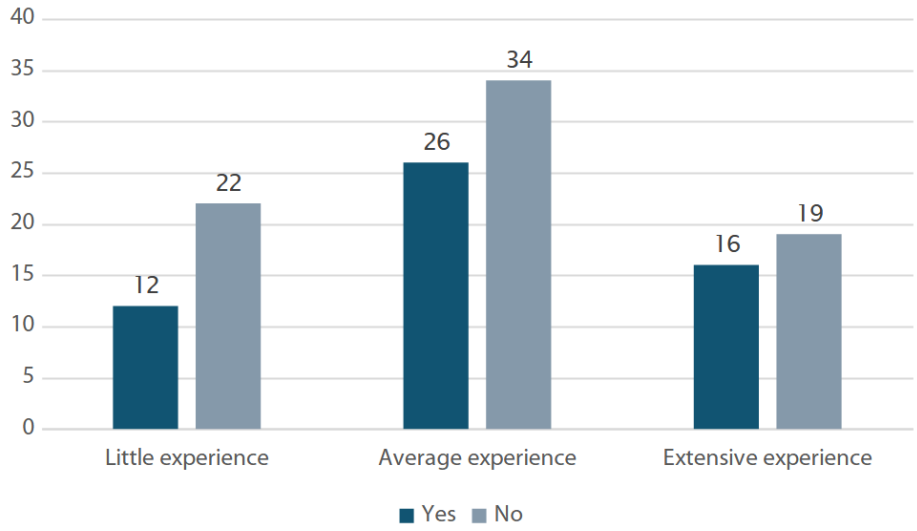


Figure 3. Reasons why GeoGebra is not used.

It is interesting to note that the teachers with extensive experience used didactic considerations to justify the lack of use of GeoGebra, while the other teacher groups rarely mentioned this type of considerations. It is also worth noting that lack of knowledge about the use of GeoGebra is a reason given at similar rates regardless of the level of experience (Figure 4).

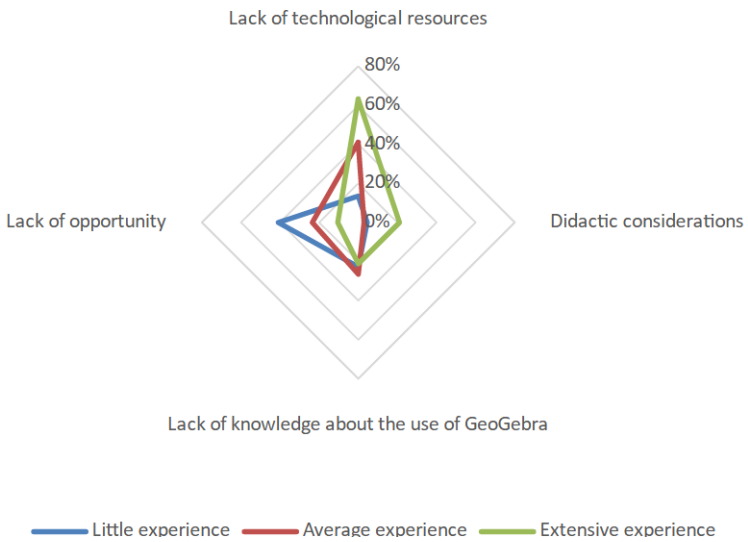


Figure 4. Reasons why GeoGebra is not used, according to level of experience.

3.2. Applet design

The number of teachers who created their own GeoGebra applets to teach the limit of a function is quite low, just 8 out of 129 participants. This means that, out of the 54 teachers who claimed to have used GeoGebra in this setting, more than 85% resorted to ready-made applets and have never designed or created a GeoGebra applet to teach the limit concept. Figure 5 shows the reasons why the teachers did not prepare their own applets.

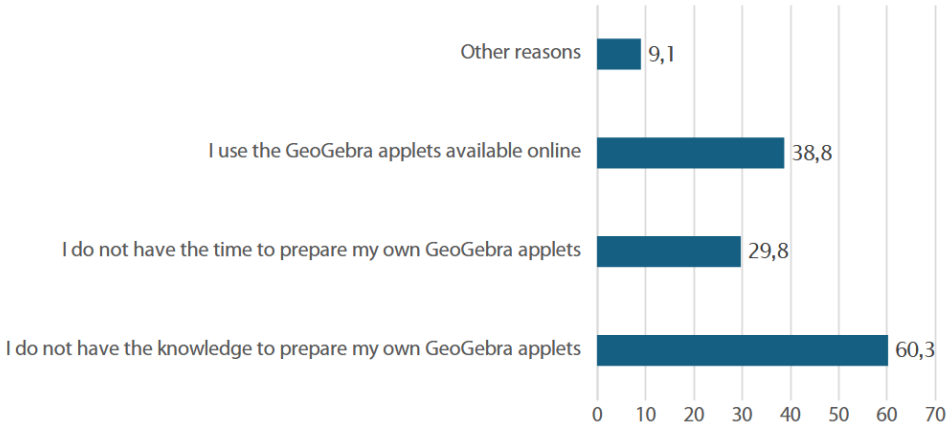


Figure 5. Reasons for not creating their own applets.

Thus, the main reason is lack of knowledge about the design and creation of GeoGebra applets (60.3%). The next most common reason is the use of GeoGebra applets that are available over the internet and, finally, the lack of time to prepare them. Other reasons given by the teachers for not preparing their own GeoGebra applets to teach the limit concept include the lack of teaching experience, lack of experience teaching this content and lack of technological resources to create and use them in the classroom.

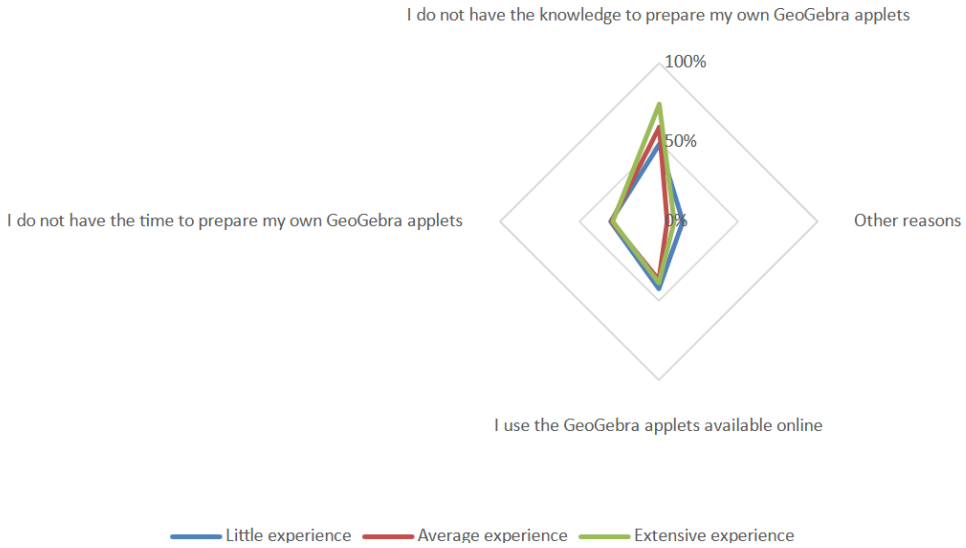


Figure 6. Reasons given for not preparing their own applets, according to level of experience.

The most striking feature of the graph above (Figure 6) is the fact that the teachers with the most extensive experience tend to assert lack of knowledge most often as the reason for not preparing their own applets. The teachers with little experience, in turn, give other reasons for not being able to create their own GeoGebra applets (such as lack of teaching experience) at a higher rate than teachers with average or extensive experience. For all the other reasons given, the percentages are virtually the same regardless of the level of experience.

3.3. Usage and usefulness

One of the questions in the questionnaire (Appendix I) addressed the importance given to the different features of the GeoGebra applets for use in teaching the limit of a function. Figure 7 shows the results obtained.

Examining this graph, it is clear that interactivity is the feature that the largest percentage of participants in this study (64.2%) ranked the highest when using the applets to teach the limit concept, followed by the possibility of combining several systems for representing the limit (54.7%). The possibility of jointly addressing several conceptual aspects of the limit appears to be less important, given that just 39.6% of the teachers decided to give it the highest score. In fact, the first two features share the same mean (5), while the mean of this latter feature of GeoGebra applets is 4. The other features that prompted the teachers to use GeoGebra applets in the process of teaching the limit concept include simplification of the process of visualising mathematics concepts for the students (in particular, the limit of a function) and ease of use.

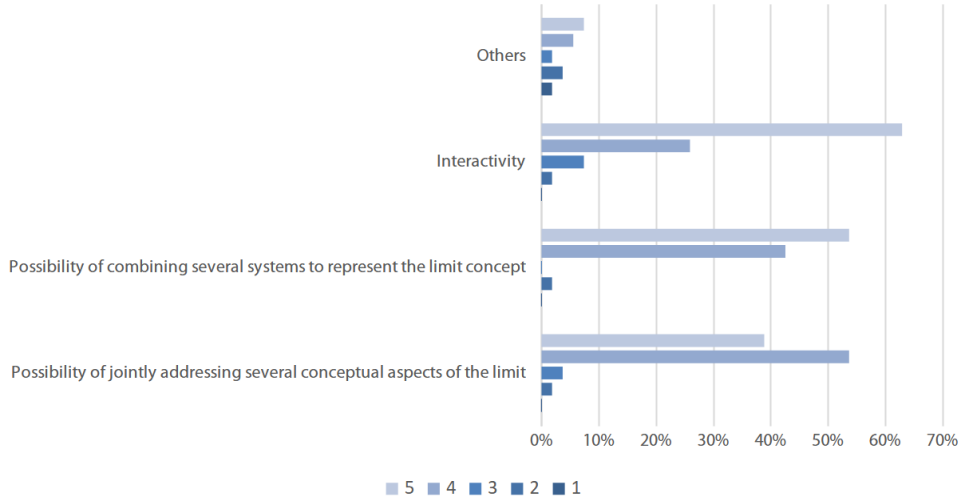


Figure 7. Importance of the applet features.

The graph below (Figure 8) shows that the percentage of teachers giving the highest score to each feature is quite similar, regardless of the level of teaching experience.

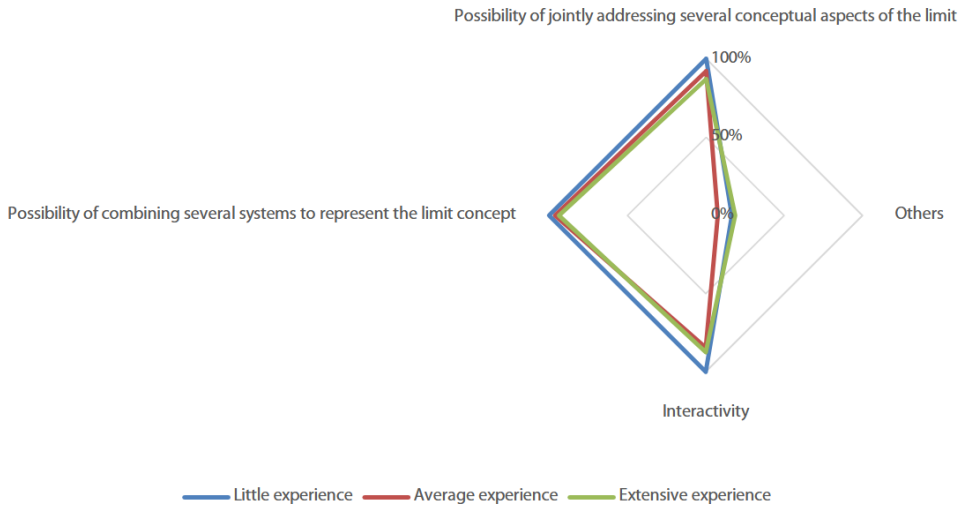


Figure 8. Percentage of highest scores according to level of experience.

There was also an open question to identify the reasons why using GeoGebra applets to teach the limit of a function was found to be useful. To analyse the responses to this question, three different classifications were made. With regard to the didactic criterion, nearly 39% of the teachers that use GeoGebra mention reasons related to teaching the limit concept, whereas more than 46% refer to issues about learning these concepts, focusing the attention on the students. Almost 30% of the teachers gave reasons that are not related to either of these two issues (Figure 9).

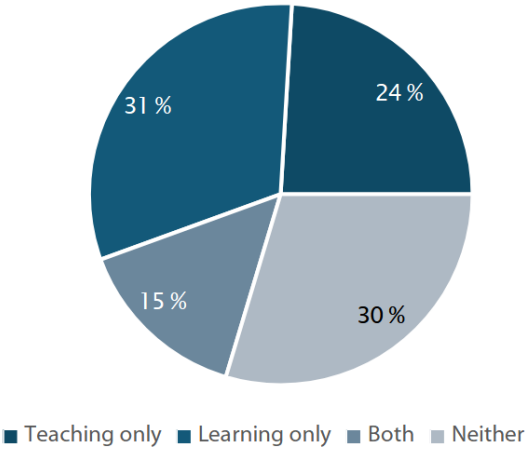


Figure 9. Didactic criteria on the usefulness of using GeoGebra.

The breakdown of the responses mentioning one or more of the main advantages of using GeoGebra has also been examined (Figure 10). The main advantage highlighted here is the visualisation capacity, recognised by more than 53% of the teachers, and how it aids in understanding the limit concept (31.5%). At the opposite end is the support in problem solving (11.1%) and emotional or motivational aspects (9.3%).

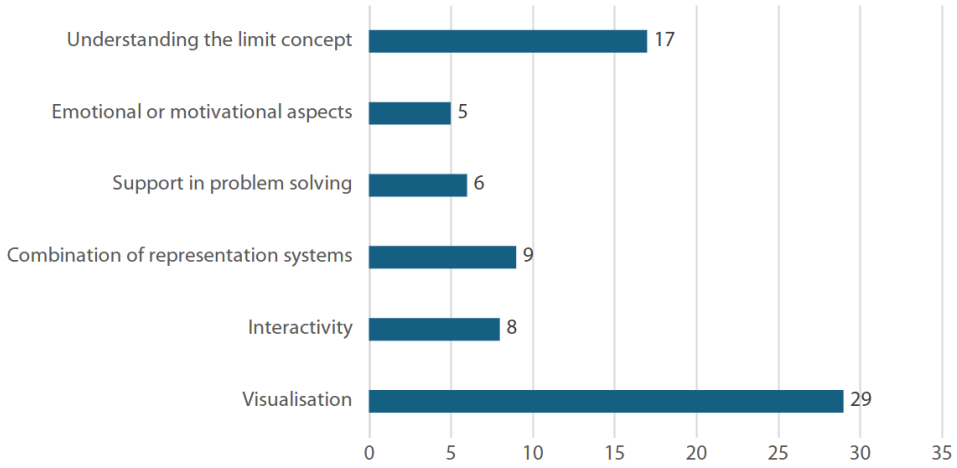


Figure 10. Advantages of using GeoGebra.

Figure 11 shows that the frequency with which these advantages of using GeoGebra are mentioned depends very little on teaching experience.

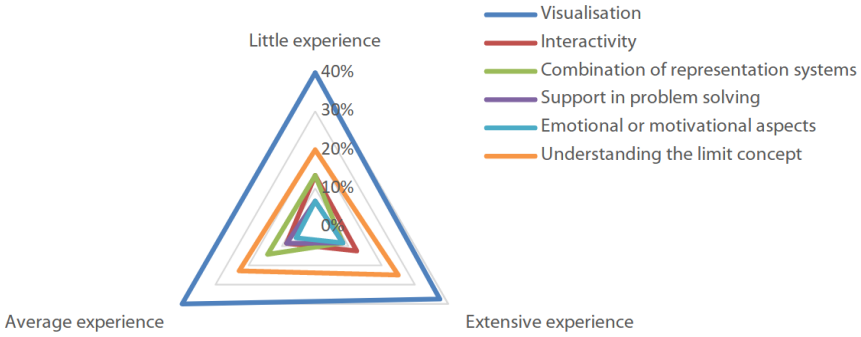


Figure 11. Advantages of using GeoGebra according to level of experience.

When it comes to the number of advantages indicated by each teacher in relation to the use of GeoGebra to teach the limit concept, none gave more than 3 advantages. The most experienced teachers tend to consider 3 simultaneous advantages more often than their colleagues (Figure 12).

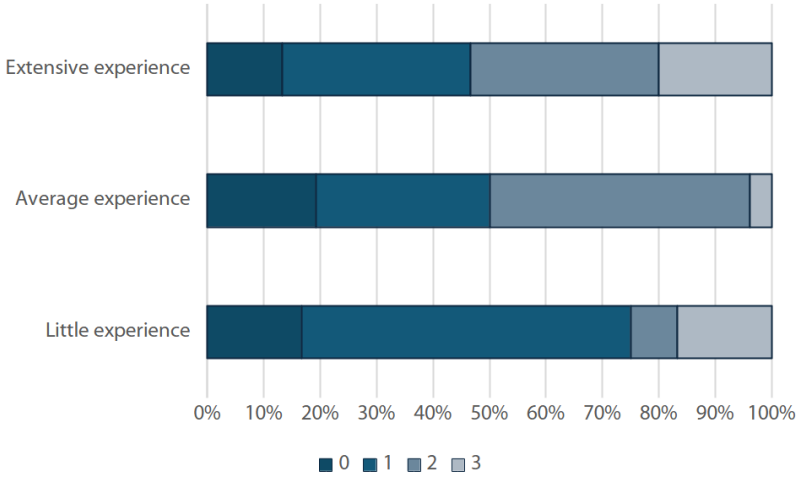


Figura 12. Número de ventajas del uso de GeoGebra según la experiencia.

Thirdly, the responses about interest in using GeoGebra to teach the limit concept were classified according to the mathematical processes defined by the NCTM. Representation ranks much more highly than the others, mentioned by more than 54% of the teachers (Figure 13). However, communication is not mentioned as a process considered by any of the teachers that use GeoGebra.

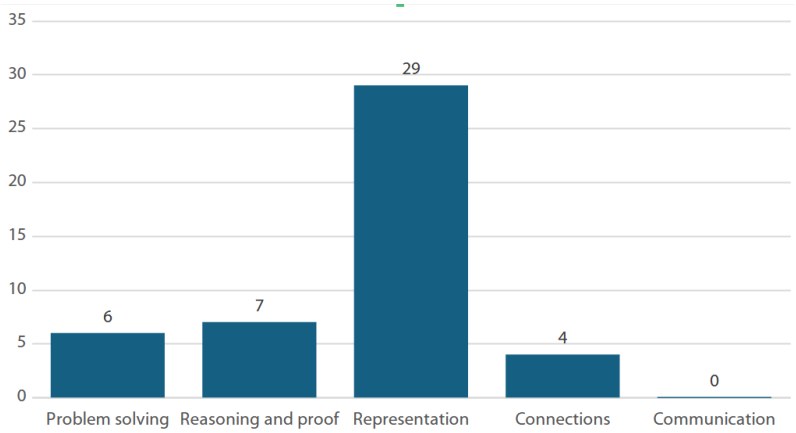


Figure 13. Mathematical processes (NCTM) considered for the use of GeoGebra.

Interestingly, teachers with little experience do not mention reasoning and proof among the processes considered. It is also striking that teachers with average experience find connections to be unimportant and those with little experience find problem solving to be less important than their colleagues, who consider it more important (Figure 14).

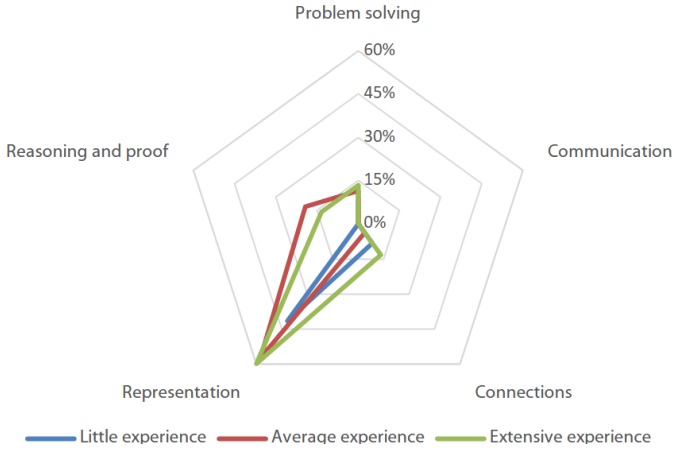


Figure 14. Mathematical processes (NCTM) considered according to level of experience.

With regard to the number of mathematical processes considered, just one teacher, with average experience, mentions 3 processes simultaneously, while more than 3 processes are not mentioned at all. On the other hand, approximately 30% of the teachers do not mention any processes (Figure 15).

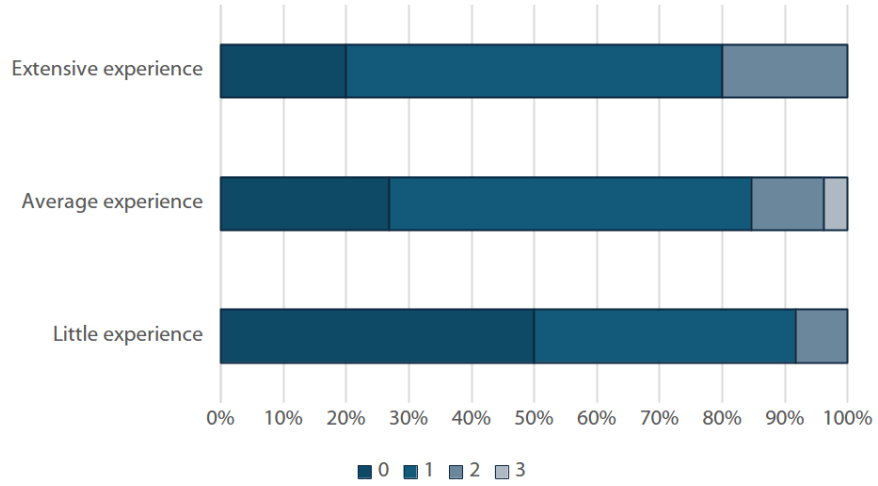


Figure 15. Number of mathematical processes (NCTM) considered according to level of experience.

3.4. Timing of use

Firstly, the participants were asked about the different stages in the process of teaching the limit concept at which the GeoGebra applets were used. The findings show that 83.3% felt that the right time to use them is when giving examples, while 77.8% use them to solve exercises. This was followed by the introduction of concepts and solving contextualised problems, while just over one quarter (25.9%) use GeoGebra applets in the evaluation (Figure 16).

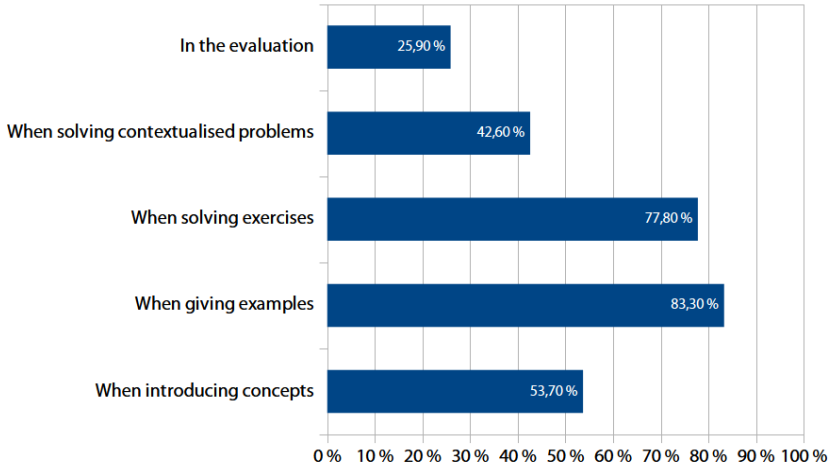


Figure 16. Timing of GeoGebra use.

The analysis of the stages in the process of teaching the limit concept at which the teachers use GeoGebra applets, according to their level of experience, shown in Figure 17 indicates that teachers with little experience rarely use this tool in the evaluation process. There is also a striking difference between the percentage of teachers with extensive experience (similar to those with little experience) and those with average experience when introducing concepts and solving contextualised problems.

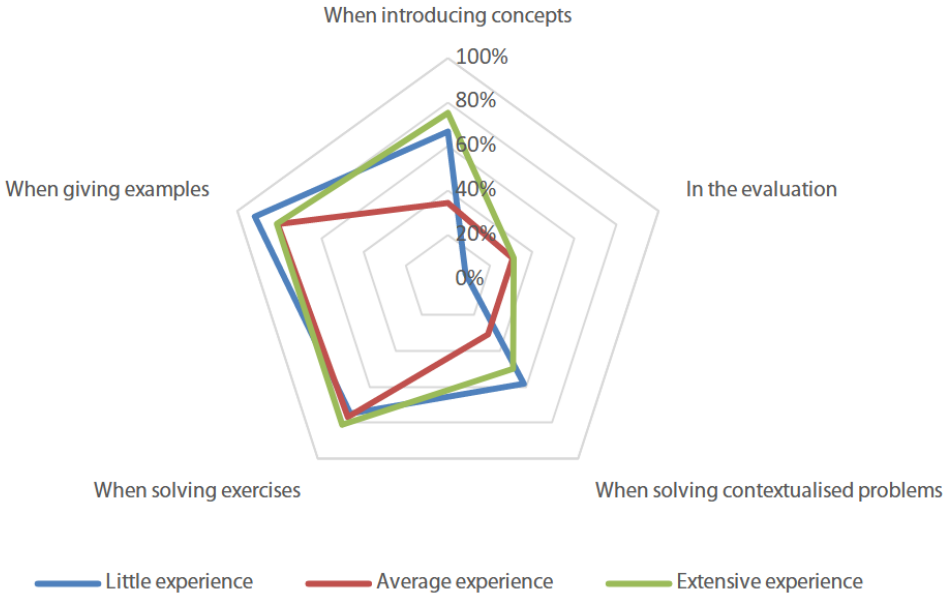


Figure 17. Timing of GeoGebra use according to level of experience.

When it comes to the stages in the process of teaching the limit concept at which this type of resource is used (Figure 18), it is worth noting the limited number of teachers who use GeoGebra applets at the five stages proposed, due to the fact that a low percentage of teachers use GeoGebra in the evaluation.

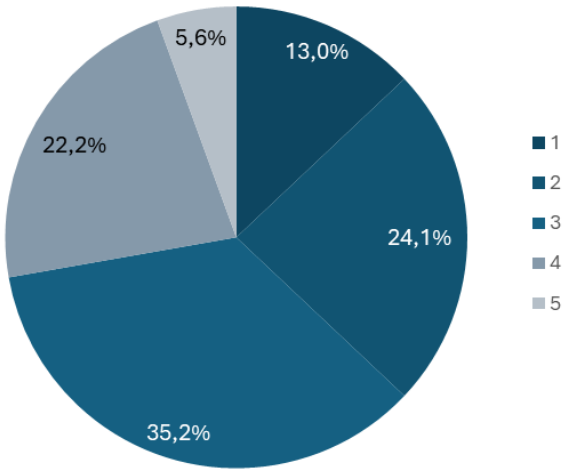


Figure 18. Number of stages at which GeoGebra is used.

However, bearing in mind the teachers' level of experience, none of those with little experience mention all five stages of the limit teaching process to introduce the use of GeoGebra. In particular, just one of these teachers uses GeoGebra in the evaluation (Figure 19).

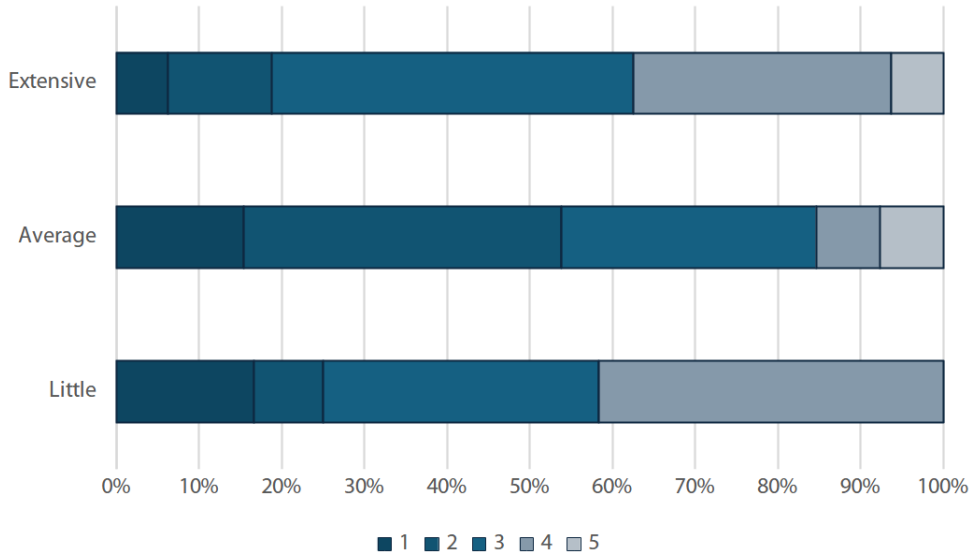


Figure 19. Number of stages at which GeoGebra is used according to level of experience.

4. Discussion and Conclusions

Just 42% of the teachers were found to have used GeoGebra in the process of teaching the limit of a function. Furthermore, the percentage of teachers that use GeoGebra in this context increases with the amount of teaching experience, although this percentage is lower in all cases than that of teachers who do not use GeoGebra in teaching the limit concept. The reasons given by those teachers that do not use GeoGebra include, most notably, the lack of technological resources, although it is worth mentioning that nearly one quarter of these teachers assert that they lack sufficient knowledge to use the tool (Wassie and Zergaw, 2019; Saralar-Aras, 2022).

In turn, the number of teachers who claim to create their own applets is extremely low (15%). The reason given for this is, in most cases, related to lack of training in use of the software (Musa et al., 2021; Saralar-Aras, 2022; Wassie and Zergaw, 2019), which highlights, in terms of the model by Koehler et al. (2013), the importance of having adequate technical knowledge (TK) as a prerequisite for proper integration of the software into teaching practice. In addition, a good number of teachers stated that they used applets available online. In this regard, Barreras et al. (2022) note that the applets available to the public often have certain shortcomings, such as limited options for representations and inefficient promotion of conceptual images or actions, as defined by Przenioslo (2004). This means that teachers need to be "critical when selecting external online resources" (Barreras et al., 2022, p. 79), evidencing the importance of moving beyond mere technological knowledge when it comes to teacher training (McGrath et al., 2011). In turn, nearly one third of the participants mentioned lack of time to prepare their own materials, a limiting factor that has also been identified in contexts such as modelling in the classroom (Schmidt, 2011) or implementation of problem-based learning (Nurlaily et al., 2019), so it is interesting to note that the different factors mentioned by the teachers seem to intersect with the introduction of diverse tools, instruments or methodologies in the classroom.

When using GeoGebra applets, the feature most highly valued by teachers is interactivity, followed by the possibility of combining several systems to represent the limit concept. Both interactivity (Sari, 2017) and the combination of several representation systems (Blázquez and Ortega, 2001) have been identified in the literature as features that can help students understand the limit concept. Furthermore, Barreras et al. (2022) note that both of these features are essential when it comes to selecting GeoGebra applets available online. It is striking that there are very few differences in the features highlighted by the teachers across the different levels of experience.

To analyse the usefulness of using GeoGebra, a three-tiered analysis was performed. Firstly, more than 46% of the teachers that use GeoGebra focus the attention on the students, indicating reasons related to learning about the limit of a function, whereas nearly 39% refer to matters related to teaching the concept. Secondly, regardless of their level of experience, the teachers highlight two main advantages in the use of GeoGebra: visualisation and comprehension of the limit concept. The striking thing in this sense is that the advantages cited least are problem solving (11%) and emotional or motivational aspects (9%), given the existence of research like that by García et al. (2021), which shows that the use of GeoGebra enhances student motivation. Thirdly, representation stands out in the analysis of the mathematical processes (NCTM) that are fostered by the use of GeoGebra. Despite the advantages found in the use of GeoGebra in processes like communication, reasoning and proof and problem solving (Romero et al., 2015), none of the surveyed teachers feel that this tool helps in the development of the communication process.

Finally, in terms of timing, the teachers stated that they used GeoGebra most often to give examples, which is related to the illustrative stage, as defined by Lasa and Wilhelmi (2013). Solving exercises, which is related to the exploratory stage, was ranked second. The demonstrative stages, which could be partially linked to the introduction of concepts, are addressed by just over one half of the participants that use GeoGebra. In addition, we believe that there could be a certain correlation between the stages at which GeoGebra is introduced in the classroom and some of the reasons indicated by McCulloch et al. (2018) for using technology in the classroom. Thus, using it to solve exercises or contextualised problems is linked to opportunities to practice, usage in introducing concepts is linked to making sense of mathematical ideas or procedures and using it to give examples is related to opportunities to build understanding. Despite the fact that some research has shown that it is possible to design valid evaluation instruments using GeoGebra (Rosyidi et al., 2024), we have observed that GeoGebra is used the least at the evaluation stage. This is consistent with studies like that by McCulloch et al. (2018), who showed that teachers do not identify GeoGebra as a technological tool for evaluation. In addition, we have seen that teaching experience is a factor that influences the idea of using GeoGebra for this purpose. This seems to suggest that training in this regard, which is related to what Koehler et al. (2013) referred to as technological pedagogical knowledge (TPK), could promote this use of GeoGebra as an evaluation tool, which could have a positive impact on student motivation (Carvalho et al., 2023).

Based on this research, the authors propose expanding the study to obtain a larger sample of teachers so that the findings are more statistically representative. Furthermore, this study is part of a research project that entails a transfer of the findings, focused on teacher training in the use of GeoGebra in secondary education.

5. Acknowledgments

This work has been partially funded by a UNIR2024 Internal Project granted to the Research Group on Didactics of Mathematics and Experimental Sciences (DIMACE) at UNIR. Additionally, the third author has been funded by the S60_23R «Research in Mathematics Education» group of the Government of Aragon.

6. References

Arnal-Bailera, A. y Oller-Marcén, A. M. (2020). Construcciones geométricas en GeoGebra a partir de diferentes sistemas de representación: un estudio con maestros de primaria en formación. Educación *Matemática, 32*(1), 67-98. https://doi.org/10.24844/EM3201.04

Barreras, A., Dubarbie, L. y Oller-Marcén, A. M. (2022). Análisis de applets de GeoGebra para la enseñanza del límite de una función. Bordón. *Revista de Pedagogía, 74*(4), 65-83. https://doi.org/10.13042/Bordon.2022.93361

Blaikie, N. (2003). *Analyzing quantitative data*. SAGE.

Blázquez, S. y Ortega, T. (2001). Los sistemas de representación en la enseñanza del límite. *Revista Latinoamericana de Investigación en Matemática Educativa*, 4(3), 219-236.

Caligaris, M. G., Schivo, M. E. y Romiti, M. R. (2015). Calculus & GeoGebra, an interesting partnership. *Procedia-Social and Behavioral Sciences*, *174*, 1183-1188. https://doi.org/10.5565/rev/ensciencias.3653

Carvalho, P., Descalço, L. y Gonçalves, H. F. (2023). Using Computer Algebra Systems in Teaching and Assessment in Calculus. En L. Gómez, C. González y J. Lees (Eds.). *EDULEARN23 Proceedings* (pp. 2041-2048). IATED.

Creswell, J. W. (2012). *Educational research. Planning, conducting and evaluating quantitative and qualitative research*. Pearson.

Dockendorff, M. y Solar, H. (2016). Formación de profesorado: conceptualización del uso del *software* GeoGebra en la enseñanza de la matemática en educación media como parte de la didáctica de la disciplina. *RECHIEM. Revista Chilena de Educación Matemática*, *10*(1), 92-99.

Elangovan, N. y Sundaravel, E. (2021). Method of preparing a document for survey instrument validation by experts. *MethodsX*, *8*, 101326. https://doi.org/10.1016/j.mex.2021.101326

Flick, U. (2004). Triangulation in qualitative research. En U. Flick, E. von Kardoff e I. Steinke (Eds.). *A companion to qualitative research* (pp. 78-183). SAGE.

García, M. M., Romero, I. M. y Gil, F. (2021). Efectos de trabajar con GeoGebra en el aula en la relación afecto-cognición. *Enseñanza de las Ciencias, 39*(3), 177-198. https://doi.org/10.5565/rev/ensciencias.3653

García-Lázaro, D. y Martín-Nieto, R. (2023). Competencia matemática y digital del futuro docente mediante el uso de GeoGebra. *ALTERIDAD. Revista de Educación*, *18*(1), 85-98. https://doi.org/10.17163/alt.v18n1.2023.07

González Pérez, A. y De Pablos Pons, J. (2015). Factores que dificultan la integración de las TIC en las aulas. *Revista de Investigación Educativa, 33*(2), 401-417. http://dx.doi.org/10.6018/rie.33.2.198161

Guarin, S. A. y Parada, S. E. (2023). Acciones y expresiones de la comprensión del límite de una función en un punto, por estudiantes de cálculo diferencial. *Educación Matemática*, 35(1), 197-228. https://doi.org/10.24844/em3501.08

Hernández, R., Fernández C. y Baptista P. (2010). *Metodología de la Investigación*. Mc Graw Hill.

Hohenwarter, J., Hohenwarter, M. y Lavicza, Z. (2009). Introducing dynamic mathematics software to secondary school teachers: the case of GeoGebra. *Journal of Computers in*

Mathematics and Science Teaching, *28*(2), 135-146.

Huang, R. y Li, Y. (2012). What matters most: A comparison of expert and novice teachers' noticing of mathematics classroom events. *School science and mathematics*, *112*(7), 420-432. https://doi.org/10.1111/j.1949-8594.2012.00161.x

Hutkemri, E. Z. (2014). Impact of using GeoGebra on students' conceptual and procedural knowledge of limit function. *Mediterranean Journal of Social Sciences*, *5*(23), 873-881. https://doi.org/10.5901/mjss.2014.v5n23p873

Iranzo, N. y Fortuny, J. M. (2009). La influencia conjunta del uso de GeoGebra y lápiz y papel en la adquisición de competencias del alumnado. *Enseñanza de las Ciencias*, *27*(3), 433-446. https://doi.org/10.5565/rev/ensciencias.3653

Koehler, M. J., Mishra, P. y Cain, W. (2013). What is Technological Pedagogical Content Knowledge (TPACK)? *Journal of Education*, *193*(3), 13–19. https://doi.org/10.1177/002205741319300303

Lasa, A. y Wilhelmi, M. R. (2013). Use of GeoGebra in explorative, explanatory and demonstrative moments. *Revista do Instituto GeoGebra Internacional de Sao Paulo*, *2*(1), 52-64.

Marange, I. Y. y Tatira, B. (2023). Teaching Euclidean geometry with GeoGebra: Perceptions for in-service mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(12), em2367. https://doi.org/10.29333/ejmste/13861

Martinovic, D. y Karadag, Z. (2012). Dynamic and interactive mathematics learning environments: the case of teaching the limit concept. *Teaching Mathematics and Its Applications: International Journal of the IMA*, *31*(1), 41-48. https://doi.org/10.1093/teamat/hrr029

McCulloch, A. W., Hollebrands, K., Lee, H., Harrison, T. y Mutlu, A. (2018). Factors That Influence Secondary Mathematics Teachers' Integration of Technology in Mathematics Lessons. *Computers & Education*, *123*(4), 26-40. https://doi.org/10.1016/j.compedu.2018.04.008

McGrath, J., Karabas, G. y Willis. J. (2011). From TPACK concept to TPACK practice: An analysis of the suitability and usefulness of the concept as a guide in the real world of teacher development. *International Journal of Technology in Teaching and Learning*, *7*(1), 1-23.

Musa, M., Mamat, Y. y Ghazali, M. (2021). Teachers' Status of GeoGebra Use in The Teaching of Geometric Transformation. *Turkish Journal of Computer and Mathematics Education*, *12*(14), 4326-4332. https://doi.org/10.17762/turcomat.v12i14.11294

NCTM (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Nurlaily, V. A., Soegiyanto, H. y Usodo, B. (2019). Elementary School Teachers' Obstacles in the Implementation of Problem-Based Learning Model in Mathematics Learning. *Journal on Mathematics Education*, *10*(2), 229-238. https://doi.org/10.22342/jme.10.2.5386.229-238

OECD (2013). ¿Qué se puede hacer para ayudar a los profesores noveles? *Teaching in Focus*, *2*.

Perrin-Glorian, M. J. (1999). A study of teachers' practice, organization of contents and of the students's work. En K. Krainer y F. Goffree (Eds.). *On research in teacher education. From a study of teaching practices to issues in teacher education* (pp. 171-186). Forschungsinstitut für Mathematikdidaktik.

Przenioslo, M. (2004). Images of the limit of function formed in the course of mathematical studies at the university. *Educational Studies in Mathematics*, *55*, 103-132. https://doi.org/10.1023/B:EDUC.0000017667.70982.05

Rodríguez, L., Bravo, J. L., Pérez, A. y Rodríguez, N. (2020). El GeoGebra como recurso didáctico para la comprensión de las formas indeterminadas del límite. *Acta Latinoamericana de Matemática Educativa*, *33*(1), 751-762.

Romero, I. M. y García, M. M. (2023). Mathematical attitudes transformation when introducing GeoGebra in the secondary classroom. *Education and Information Technologies*, 1-26. https://doi.org/10.1007/s10639-023-12085-w

Romero, I. M., García, M. M. y Codina, A. (2015). Developing mathematical competencies in secondary students by introducing dynamic geometry systems in the classroom. *Education and Science*, *40*(177), 43-58. http://dx.doi.org/10.15390/EB.2015.2640

Rosyidi, A. H., Sari, Y. M., Fardah, D. K. y Masriyah, M. (2024). Designing mathematics problem-solving assessment with GeoGebra Classroom: proving the instrument validity. *Journal of Education and Learning (EduLearn)*, *18*(3), 1030-1038. https://doi.org/10.11591/edulearn.v18i3.21191

Saralar-Aras, I. (2022). An exploration of middle school mathematics teachers' beliefs and goals regarding a dynamic tool in mathematics lessons: Case of GeoGebra. *Journal of Research in Science, Mathematics and Technology Education*, *5*(SI), 41-63. https://doi.org/10.31756/jrsmte.113SI

Sari, P. (2017). GeoGebra as a means for understanding limit concepts. *Southeast Asian Mathematics Education Journal*, *7*(2), 71-84. https://doi.org/10.46517/seamej.v7i2.55

Schmidt, B. (2011). Modelling in the classroom: Obstacles from the teacher's perspective. En G. Kaiser, W. Blum, R. Borromeo Ferri y G. Stillman (Eds.). *Trends in teaching and learning of mathematical modelling: ICTMA14* (pp. 641-651). Springer. https://doi.org/10.1007/978-94-007-0910-2_61

Takaci, D., Stankov, G. y Milanovic, I. (2015). Efficiency of learning environment using GeoGebra when calculus contents are learned in collaborative groups. *Computers & Education*, *82*, 421-431. https://doi.org/10.1016/j.compedu.2014.12.002

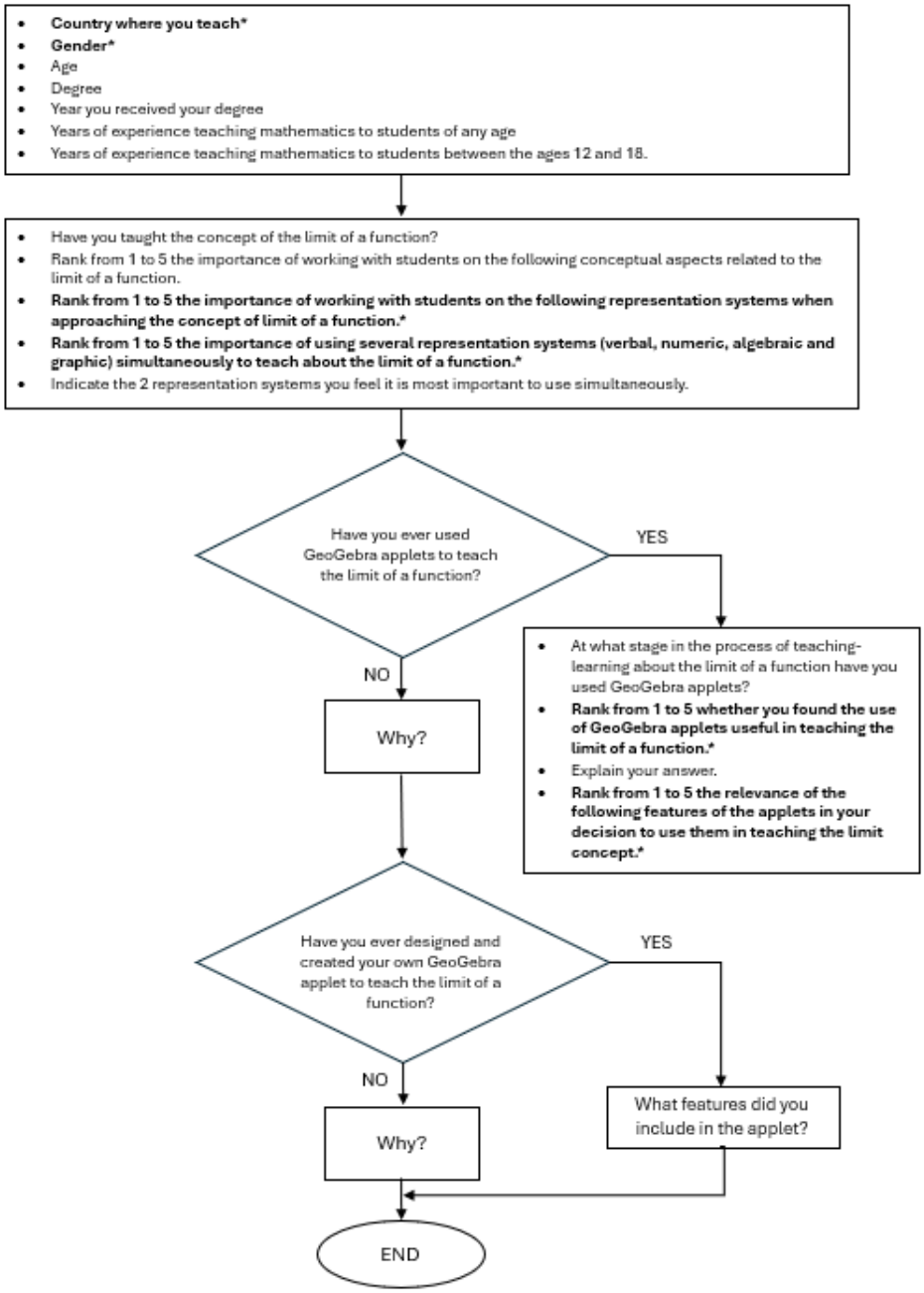
Tall, D. (1991). The Psychology of Advanced Mathematical Thinking. En D. Tall (Ed.). *Advanced Mathematical Thinking* (pp. 3-21). Kluwer. https://doi.org/10.1007/0-306-47203-1_1

Wassie, Y. A. y Zergaw, G. A. (2019). Some of the potential affordances, challenges and limitations of using GeoGebra in mathematics education. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(8), em1734. https://doi.org/10.29333/ejmste/108436

Zengìn, Y. (2017a). The potential of GeoGebra software for providing mathematical communication in the light of pre-service teachers' views. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *11*(1), 101-127.

Zengìn, Y. (2017b). The effects of GeoGebra software on preservice mathematics teachers' attitudes and views toward proof and proving. *International Journal of Mathematical Education in Science and Technology*, *48*(7), 1002-1022. https://doi.org/10.1080/0020739X.2017.1298855

APPENDIX I. Questionnaire flow chart.



* Questions modified according to expert opinions.